OSNOVNI TRIGONOMETRIJSKI IDENTITETI

$1)\ \sin^2\alpha+\cos^2\alpha=1$

$2)\ tg\alpha=\dfrac{\sin\alpha}{\cos\alpha}$

$3)\ ctg\alpha=\dfrac{\cos\alpha}{\sin\alpha}$

$4)\ tg\alpha\cdot ctg\alpha=1$

Da probamo da dokažemo neke od indetiteta:

1) $\sin^2\alpha+\cos^2\alpha=$(pogledajmo definicije: $\sin\alpha=\dfrac{a}{c}$ i $\cos\alpha=\dfrac{b}{c}$ to da zapamtimo)=

$\dfrac{a^2}{c^2}+\dfrac{b^2}{c^2}=\dfrac{a^2+b^2}{c^2}=$(važi Pitagorina teorema, $a^2+b^2=c^2$)$=\dfrac{c^2}{c^2}=1$

2) $\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\frac{a}{c}}{\frac{b}{c}}=\dfrac{a\cdot c}{b\cdot c}=\dfrac{a}{b}=tg\alpha$ slično se dokazuje i za $ctg\alpha$

4) $tg\alpha\cdot ctg\alpha=$(zamenimo iz definicije, da je $tg\alpha=\dfrac{a}{b}$ i $ctg\alpha=\dfrac{b}{a}$)$=\dfrac{a}{b}\cdot\dfrac{b}{a}=1$

Iz osnovnih indetiteta se mogu izvesti razne druge jednakosti:

1) Ako krenemo od:

$\sin^2\alpha+\cos^2\alpha=1\rightarrow$ ovo delimo sa $\cos^2\alpha$

$$\frac{\sin^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}$$

$tg^2\alpha+1=\dfrac{1}{\cos^2\alpha}\rightarrow$ Odavde izrazimo $\cos^2\alpha$

$$\boxed{\cos^2\alpha=\frac{1}{tg^2\alpha+1}}$$

Ako sad ovo zamenimo u:

$$\sin^2\alpha+\cos^2\alpha=1$$

$$\sin^2\alpha+\frac{1}{tg^2\alpha+1}=1$$

$$\sin^2\alpha=1-\frac{1}{tg^2\alpha+1}$$

$$\sin^2\alpha = \frac{tg^2\alpha + 1 - 1}{tg^2\alpha + 1}$$

$$\boxed{\sin^2\alpha = \frac{tg^2\alpha}{tg^2\alpha + 1}}$$

Ove dve identičnosti ćemo zapisati i koristiti ih u zadacima!!!

Zadatak 1. Dokazati identitet $\left(1 + tgx + \frac{1}{\cos x}\right)\cdot\left(1 + tgx - \frac{1}{\cos x}\right) = 2tgx$

$$\left(1 + tgx + \frac{1}{\cos x}\right)\cdot\left(1 + tgx - \frac{1}{\cos x}\right) = \left(1 + \frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)\cdot\left(1 + \frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) =$$

$$\frac{\cos x + \sin x + 1}{\cos x}\cdot\frac{\cos x + \sin x - 1}{\cos x} = \text{ gore je razlika kvadrata}$$

$$\frac{(\cos x + \sin x)^2 - 1^2}{\cos^2 x} = (\text{jedinicu ćemo zameniti sa } \sin^2 x + \cos^2 x\ )$$

$$\frac{\cos^2 x + 2\cos x\sin x + \sin^2 x - \sin^2 x - \cos^2 x}{\cos^2 x} = \frac{2\cos x\sin x}{\cos^2 x} =$$

$$= 2\frac{\sin x}{\cos x} = 2tgx$$

Zadatak 2. Dokazati identitet $\frac{3}{1 - \sin^6\alpha - \cos^6\alpha} = (tg\alpha + ctg\alpha)^2$

$$\frac{3}{1 - \sin^6 x - \cos^6 x} = \frac{3}{1 - (\sin^6 x + \cos^6 x)} = \text{ Pokušaćemo da transformišemo izraz}$$

$\sin^6 x - \cos^6 x$ Podjimo od $\sin^2 x - \cos^2 x = 1$ pa ''dignemo'' na treći stepen:

$$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$\sin^2 x + \cos^2 x = 1 /()^3$$

$$\sin^6 x + 3\sin^4 x\cos^2 x + 3\sin^2 x\cos^4 x + \cos^6 x = 1$$

$$\sin^6 x + 3\sin^2 x\cos^2 x\underbrace{(\sin^2 x + \cos^2 x)}_{1} + \cos^6 x = 1$$

Dakle: $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x\cos^2 x$

Vratimo se u zadatak:

$$= \frac{3}{1 - 1 + 3\sin^2 x\cos^2 x} = \frac{3}{3\sin^2 x\cos^2 x} = \frac{1}{\sin^2 x\cos^2 x}$$

Da vidimo sad desnu stranu:

$$(tg\alpha + ctg\alpha)^2 = tg^2\alpha + 2tg\alpha ctg\alpha + ctg^2\alpha$$

$$= \frac{\sin^2\alpha}{\cos^2\alpha} + 2 + \frac{\cos^2\alpha}{\sin^2\alpha}$$

$$= \frac{\sin^4\alpha + 2\sin^2\alpha\cos^2\alpha + \cos^4\alpha}{\sin^2\alpha\cos^2\alpha}$$

$$= \frac{(\sin^2\alpha + \cos^2\alpha)^2}{\sin^2\alpha\cos^2\alpha}$$

$$= \frac{1}{\sin^2\alpha\cos^2\alpha}$$

Ovim smo dokazali da su leva i desna strana jednake:

Uslov je $1 - \sin^6\alpha - \cos^6\alpha \neq 0$

$$\sin^6\alpha - \cos^6\alpha \neq 1$$

$$1 - 3\sin^2\alpha\cos^2\alpha \neq 1$$

$$\sin^2\alpha\cos^2\alpha \neq 0$$

$$\sin\alpha \neq 0 \wedge \cos\alpha \neq 0$$